Rate Shocks in Value-Based Measures of Interest Rate Risk

By Michael R. Arnold and Dai Zhao

Relying on rate shocks as the basis for measuring value-based interest rate risk may understate risk.

Deterministic interest rate shocks are widely used as the basis for measurement and subsequent management of both earnings- (EBIRR) and value-based interest rate risk (VBIRR). Shock-based measures are consistent with banking regulations governing the control of interest rate risk (IRR), which have also emphasized the importance of using rate shocks to measure IRR in stress-based conditions for both EBIRR and VBIRR.

When applied to EBIRR, rate-shock methodologies can provide a sufficiently accurate measure to serve as the basis for risk management and regulation. The size of the rate shock is inversely related to its likelihood, so that large rate shocks (in absolute-value terms) capture income risks at the tails of the risk distributions. It is also widely and correctly understood that larger rate shocks are required to measure risk over the longer time horizons used to measure earnings.

By contrast, value is based on discounting future cash flows extending across all time periods of a financial instrument's maturity.[1] The tail risk of instruments with short duration can be calculated using relatively small shocks, while the tail risk of instruments with long duration can only be calculated with much larger rate shocks. Consequently, the likelihood of the risk cannot be inferred directly from the likelihood of the rate shock.

Hedged or partially hedged instruments and balance sheets pose a conceptual paradox for risk mangers and governing committees. Successful hedging of risk is designed to reduce it. However, we find that rate-shock methodologies applicable to reducing VBIRR can result in material understatement of risk and may be subject to misinterpretation. We demonstrate that—even when risk is hedged against rate shocks that are perceived as unlikely—a significant likelihood of large unhedged exposures may remain. Because risk measured with unlikely and large rate shocks is perceived to be low, risk analysts may incorrectly assume that risk is controlled. We show by example a significant likelihood that large exposures can remain in balance sheets and measures of economic value of equity (EVE)-at-risk—even in relatively well-hedged portfolios.

This article has four parts:

- We compute historical rate distributions from monthly short- and long-term Treasury rate histories to illustrate that even crude measures of rate movements show interest rate changes increase with the time horizon.
- We compute VBIRR for a selection of financial instruments using the common "four-step rate-shock method" (*see* box on page **<<XX>>**) and compare it to the distribution of net present values (NPVs) calculated using a proprietary stochastic-rate-generation system.
- We "hedge" representative instruments and demonstrate that there is significant likelihood of worse results than indicated from risk results based on rate shocks.
- We combine financial instruments to represent the risks of a balance sheet in order to demonstrate that the reported results hold for EVE-at-risk measures.

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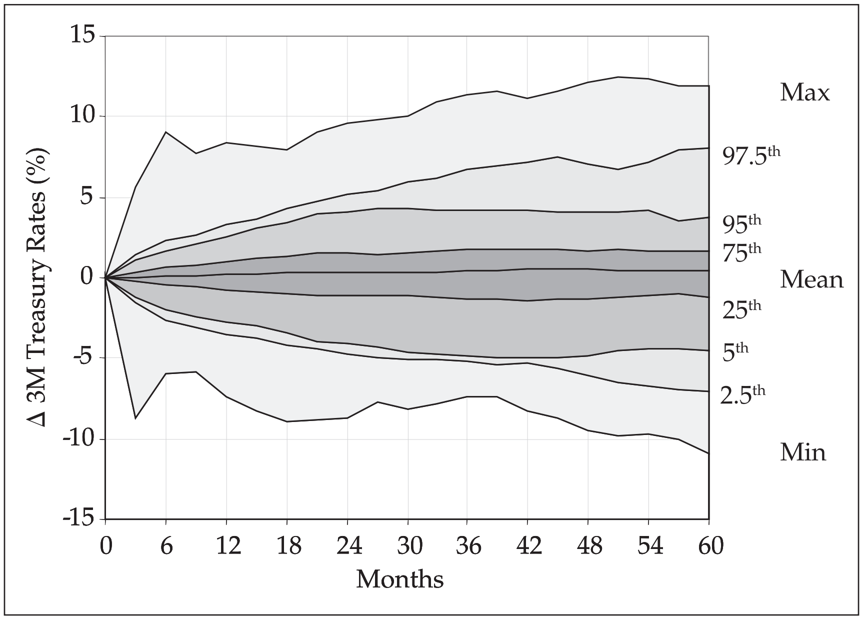
Exhibit 1. Historical Distribution of Short-Term Rate Changes



In the contour graph shown in Exhibit 1, the change of the 3M Treasury rate over sequential intervals is used to calculate the cumulative distribution over sequential intervals. These results are stacked in such a way to produce a graphical representation of rate changes. The percentiles are labeled on the right. The solid line in the middle represents the median change in rates over the indicated time periods. The large extreme rate changes under six months are associated with rate changes during 1979–1982.

Exhibit 2. Rate Changes for Selected Rates, Time Periods and Percentiles (in bps)

Rate	Direction	Percentile	Time Horizon				
			1 Year	2 Year	3 Year	4 Year	5 Year
3M Treasury	Down	Minimum	–737	–824	–888	–884	–876
		1%	–463	–463	–581	–635	–614
		2.5%	–353	–374	–415	–446	–470
	Up	97.5%	329	362	433	476	514
		99%	418	493	568	600	616
		Maximum	834	818	791	909	955
5-Year Treasury	Down	Minimum	–503	–571	–585	–622	–621
		1%	–457	–469	–487	–503	–507
		2.5%	–380	–402	–459	–472	–470
	Up	97.5%	226	270	323	316	366
		99%	341	338	398	459	478
		Maximum	526	672	512	557	652

Historical Rate Distribution

Historical rate-change histories provide support for the common belief that the likelihood of a particular rate change increases with time. We computed the empirical cumulative distributions for sequential time periods and estimate the size of rate changes at the 1st, 2.5th, 97.5th, and 99th percentiles, using monthly secondary three-month and five-year Treasury rates.[2] The three-month changes are plotted in Exhibit 1, using the contour method described in Arnold and Hawkins.[3] The rate history was downloaded from the Federal Reserve Board's (FRB's) Web site[4] and begins in January 1947. The distribution of short-term rate changes shows a distinct increase over a 60-month time frame.[5]

How likely is a particular rate change? In Exhibit 2, we compute the up-and-down rate shocks associated with selected percentiles for annual periods, using the three-month and five-year Treasury rates as *indicators*. The results show, for example, that 95 percent of the calculated changes in three-month rates over a 12-month period were between –353 and +329 basis points (bps); over three years, 95 percent of the calculated changes were between –415 and + 433 bps. For five-year rates, 95 percent of the calculated changes over 12 months were between –380 and +226 bps; over three years, changes in the five-year rates were between –459 and +323 bps.

This analysis supports the Office of the Comptroller of the Currency (OCC) guidance for rate shocks in EBIRR analyses, which states, "the OCC … encourages banks to employ 'stress tests' that consider changes of 400 basis points or more over a one-year horizon."[6] Based on this empirical analysis, a +/–400 bps rate shock[7] would be near the tail of the rate-shock distribution.

Below, we show that this paradigm is robust in VBIRR measurement, principally because the size of the shock required to assess where a risk is located on the distribution is a function of *product characteris-*

Exhibit 3. Risk-Shock Results for Selected Financial Instruments

Product	Duration	Conv	WAL	Price	Percentile in Rate Shock							
					–400 bps	–300 bps	–200 bps	–100 bps	+100 bps	+200 bps	+300 bps	+400 bps
					8.86	7.73	6.08	3.54	–4.40	–9.30	–14.43	–19.52
					na	na	3%	68%	81%	86%	91%	93%
Savings	–3.24	–0.29	5.86	–90.53	–19.54	–13.26	–7.86	–3.39	3.10	5.66	7.24	7.35
					na	100%	90%	64%	27%	19%	14%	14%
5x1 hybrid	1.86	–0.45	3.43	102.06	4.24	3.41	2.71	1.64	–2.09	–4.46	–7.00	–9.57
					7%	11%	16%	28%	85%	91%	95%	97%
Structured note	3.97	–0.86	3.83	96.51	8.86	7.73	6.08	3.54	–4.40	–9.30	–14.43	–19.52
					na	na	3%	68%	81%	86%	91%	93%
5-year bullet	4.43	0.23	5.00	98.82	19.71	14.40	9.34	4.55	–4.32	–8.42	–12.31	–16.01
					na	0%	6%	30%	77%	88%	93%	96%
Cap 10-year (long)	–1.52	0.65	10.00	2.07	–2.07	–2.05	–1.83	–1.20	1.85	4.47	8.01	12.23
					44%	42%	36%	28%	15%	10%	6%	3%
5-year floor (long)	0.72	0.66	5.00	0.67	12.41	7.51	3.48	1.05	–0.39	–0.54	–0.61	–0.64
					na	0%	4%	14%	37%	43%	48%	51%
Swap 10-year (rec fix)	7.96	0.76	10.00	0.30	38.80	27.65	17.53	8.34	–7.58	–14.48	–20.76	–26.49
					na	2%	14%	36%	72%	81%	88%	92%

Note: Results are rank-ordered from benefit to risk. na—outside the range of the stochastic risk distribution.

tics, whereas in EBIRR it is merely a function of *time period* for which earnings is analyzed.

Rate Shock and VBIRR Distributions for Financial Instruments

We use ZM Financial Systems' ALM model, which contains both lattice and Monte Carlo stochastic rate-generation technologies, to measure the distribution of NPV associated with the market prices[8] for a representative selection of financial instruments. We report results below for a representative selection of eight instruments:[9]

- MBS-FNCL 5.5
- Five-year bullet
- Savings (five-year)
- Ten-year cap (long)
- 5x1 hybrid
- Five-year floor (long)
- Five-year structured note
- Five-year swap (receive fixed)

The value distribution for each product is calculated using 2,000 rate scenarios. Then in a second procedure, the "four-step" VBIRR procedure is invoked to calculate risk in the various shock cases (for detail, *see* box on page **<<XX>>**). The results are compared by overlaying the rate-shock results on top of the distributional results.[10]

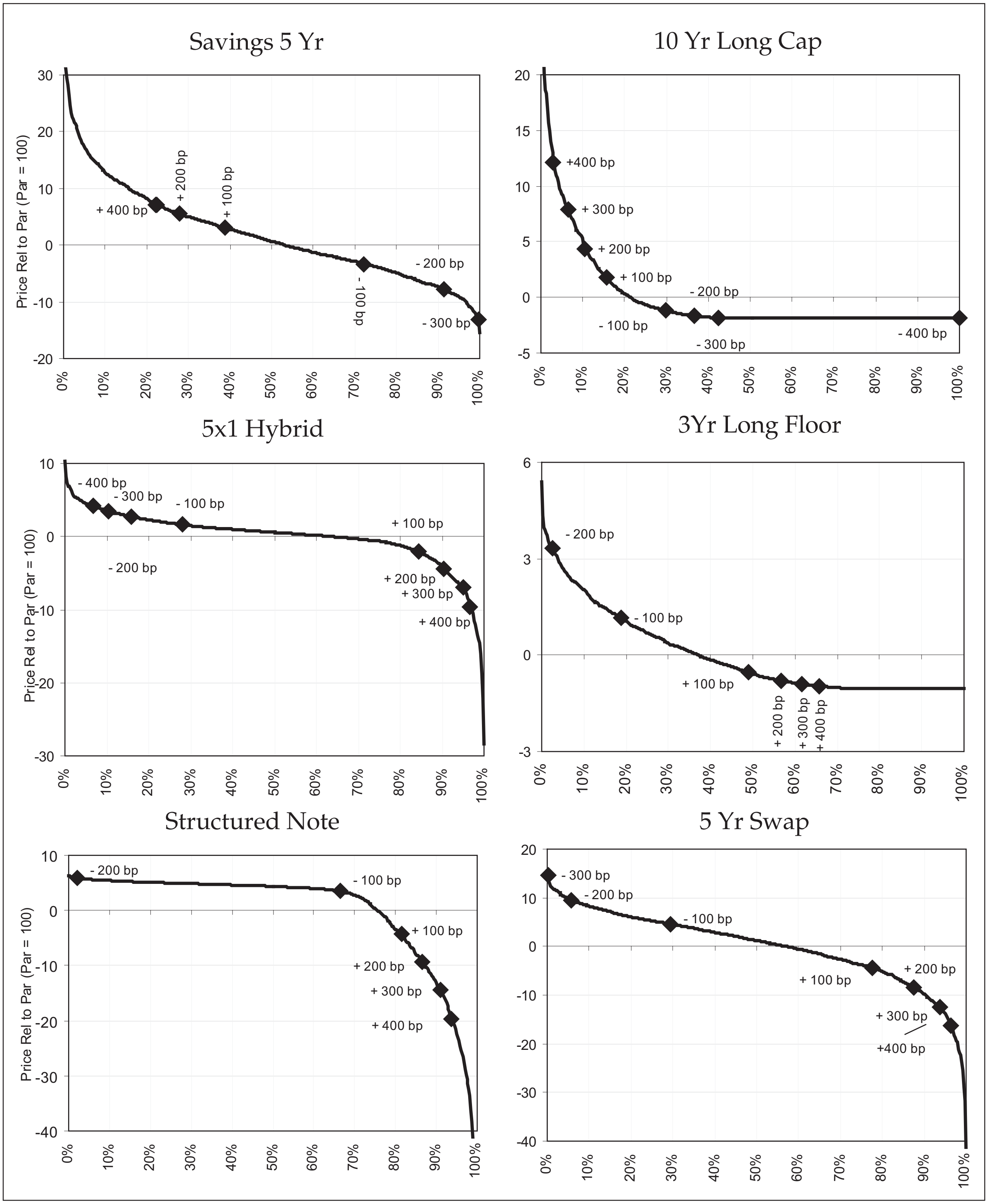
Exhibit 3 provides results for the eight products, which are graphed in Exhibit 4. The calculations show several patterns:

- Rate shocks are usually sequentially ordered.[11]
- The percentile location of the rate shocks for each instrument's risk distribution varied by product.
- Some rate-shock results were outside the range of results generated by the 2,000 stochastic results. This pattern appears to be correlated to instrument duration.
- Inference as to where a particular rate shock result may occur on an instrument's risk distribution does not appear to be practical and requires estimation.

Impact of Hedging Products

In this section, we employ the Excel Solver to find hedge solutions for selected products, targeting minimization of the sum of squared errors (SSR1) associated with the eight rate shocks.[12] We make no claims that our hedges are perfect nor that better

Exhibit 4. Graphs of Risk Distributions and Rate-Shock Measures for Eight Instruments



In the graphs above, the line represents the distribution of outcomes from the 2,000 stochastically generated scenarios; the diamonds are the results from the rate shocks.

hedges aren't available. Our purpose in this section is to demonstrate a key finding:

> Reliance on rate shocks alone appears to yield misleading results when compared with a distribution of outcomes employing the same hedge. Significant risk may result that it is not measured.

We explore three products:

- 5x1 hybrid hedged with a five-year, pay-fixed swap, three-year floor
- MBS-FNCL 5.5 hedged with combination of five-year, pay-fixed swap and three-year cap
- Money market hedged with a five-year, receive-fixed, swap, three-year floor

In Exhibit 5, we present some of the results numerically and also report the hedged risk at the 97.5th percentile.[13] Exhibit 6 displays graphical results for these three products.

A review of the information in Exhibits 5 and 6 indicates the following:

- *The hedges significantly reduce risk* (as indicated graphically) whether one uses the rate shocks or 2,000 rate-path distributions to measure risk reduction.
- *The rate shock results are no longer sequentially ordered* and appear to occur more toward the center of the distribution of outcomes.
- A significant percentage of *hedged outcomes had more risk than the largest risk reported from the rate-shock hedge*. This can be seen by looking for the highest percentile in the hedged rows. (Results are summarized in Exhibit 7.)

Larger rate shocks are required to measure risk over the longer time horizons used to measure earnings.

Comparisons with Hedges Designed to Minimize VBIRR in Stochastic Scenarios

Hedging on the basis of a distribution of 2,000 outcomes reduces the performance of hedging, as measured by risk in the eight parallel rate shocks. To illustrate, we compare the three measures side-by-side (Exhibit 8): the sum of squared errors from the eight rate shocks (SSR), the variance explained[14] (VE) and risk at the 97.5th percentile in the 2,000 paths. It should be noted that the improvement in the distributional measures is relatively small, even when there are substantial decreases in the effectiveness of the hedge, as measured in the rate shocks.

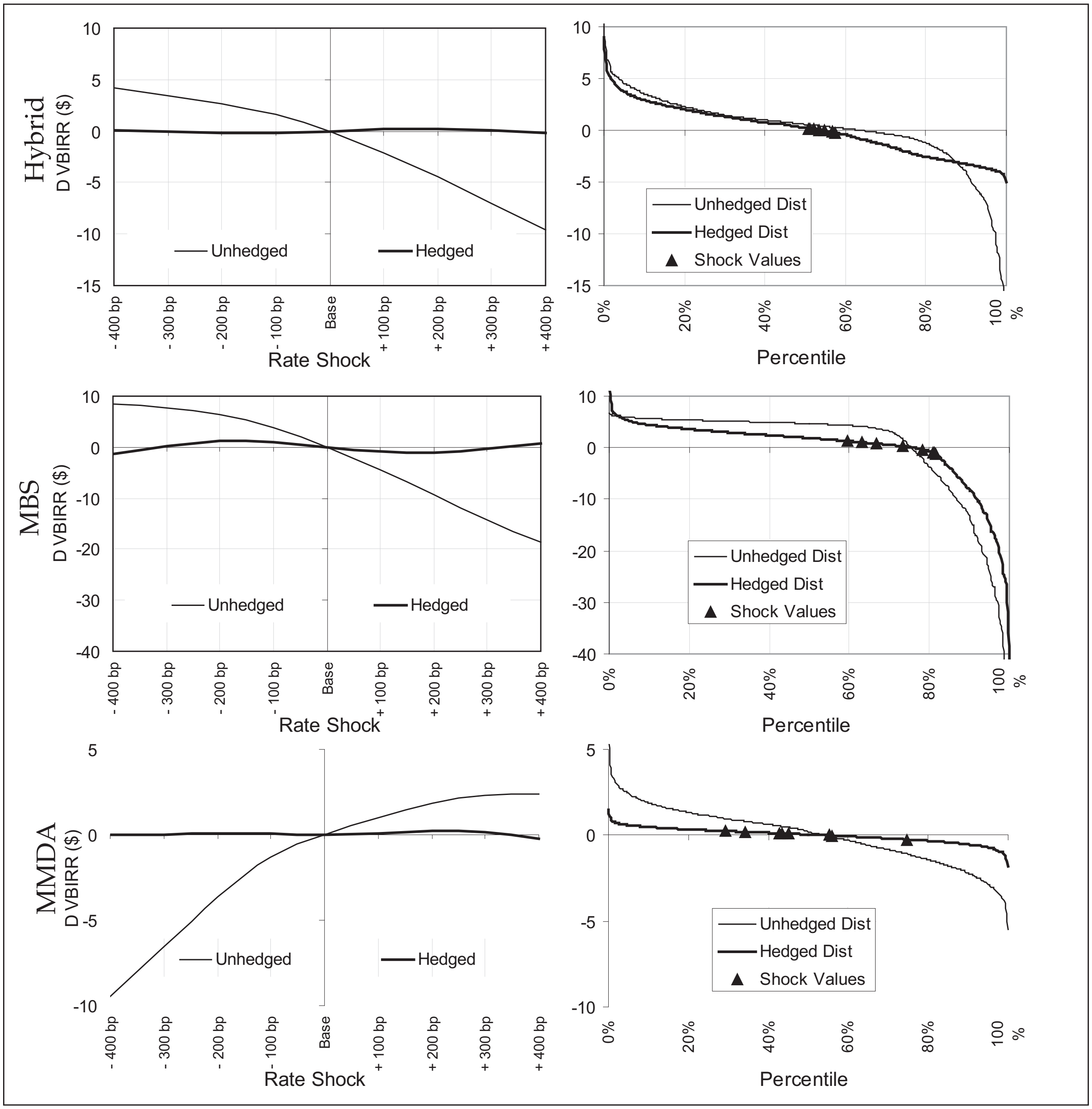
The results of this case study confirm that there are differences in the hedging results but that these differences may be minor. However, our results raise the following question: Absent testing, how can one be certain? The primary finding is that reliance on rate shocks negatively affects risk measurement *transparency*. Additional risk may be present but is

Exhibit 5. Unhedged and Hedged Risk Results for Three Products

Product	Hedged	Risk in Rate Shocks								Risk @ 97.5%
		–400 bps	–300 bps	–200 bps	–100 bps	+100 bps	+200 bps	+300 bps	+400 bps	
Hybrid	No	4.2	3.4	2.7	1.6	–2.1	–4.5	–7.0	–9.6	–10.5
		7%	11%	16%	28%	85%	91%	95%	97%	
	Yes	0.1	0	–0.2	–0.2	0.2	0.2	0	–0.2	–3.9
		53%	54%	57%	57%	51%	51%	54%	57%	
MBS	No	8.4	7.6	6.3	3.8	–4.6	–9.4	–14.1	–18.7	–32.0
		6%	11%	21%	57%	76%	84%	89%	93%	
	Yes	–1.2	0.3	1.3	1.1	–0.9	–1.0	–0.3	0.8	–20.0
		82%	73%	60%	63%	81%	81%	78%	67%	
MMDA	No	–9.4	–6.5	–3.6	–1.3	1.0	1.8	2.3	2.4	–3.4
		na	na	98%	77%	52%	28%	12%	7%	
	Yes	0.0	0.0	0.1	0.1	0.1	0.2	0.2	–0.3	–0.9
		56%	55%	45%	43%	44%	29%	34%	75%	

* Higher number in the cells is the risk in dollars. Lower number in the cells is the percentile.

Exhibit 6. Unhedged and Hedged Results for Three Products



The graphs on the left side of the page depict the shock results (unhedged and hedged) without reference to the results from the 2,000 outcomes.

The graphs on the right side depict the unhedged and hedged *distributions*, where the hedge is determined by minimizing VBIRR as measured using parallel rate shocks. This latter distribution is overlayed with the hedge results from the left hand side and depicted with the triangles.

Exhibit 7. Percentage of Outcomes with More Risk Than Occurred in Parallel Rate Shocks

Product	Percentage of Outcomes with More Risk than Worst Reported Outcome
Hybrid	43%
MBS	18%
MMDA	25%

Exhibit 8. Comparison of Hedge Results

Measure	Hedge Design	Product: Hybrid	Product: MBS	Product: MMDA
SSR in shock	Shocks	0.2	6.9	0.2
	2,000 scenarios	60.2	222.4	1.6
Risk @ 97.5% in distribution	Shocks	–3.9	–20.0	–0.9
	2,000 scenarios	–3.5	–15.0	–0.8
Variance explained in distribution	Shocks	62%	63%	94%
	2,000 scenarios	69%	73%	95%

Exhibit 9. Balance Sheets Used to Evaluate EVE-at-Risk

Balance Sheet 1	
Assets	Liability & Equity
MBS 100	MMKT 90
	Equity 10

Balance Sheet 2	
Assets	Liability & Equity
MBS1 25	MMKT 20
MBS2 25	Savings 10
Bullet 45	Checking 10
Cash 5	Bullet 50
	Equity 10

Exhibit 10. Hedging Results of Balance Sheets in Exhibit 9

Balance Sheet	Hedges	Risk in Rate Shocks –400 bps	–300 bps	–200 bps	–100 bps	+100 bps	+200 bps	+300 bps	+400 bps	Risk @ 97.5%
1	None	–0.1	1.7	3.1	2.6	–3.6	–7.7	–12.0	–16.6	–29.2
		77%	75%	67%	71%	81%	86%	90%	92%	
	Parallel	–0.3	0.3	0.4	0.1	0.2	0.3	0.2	–0.2	–13.9
		55%	49%	49%	51%	50%	49%	50%	54%	
	Parallel + non-parallel	0.0	0.3	–0.2	–0.5	0.6	0.8	0.4	–0.7	–10.8
		47%	46%	49%	51%	43%	42%	45%	53%	
2	None	–3.6	–1.6	–0.2	0.4	–1.0	–2.3	–3.9	–5.7	–6.0
		93%	81%	60%	50%	73%	87%	94%	97%	
	Parallel	–0.2	0.3	0.1	0.0	0.1	0.2	0.1	–0.1	–3.4
		60%	49%	52%	55%	53%	52%	53%	58%	
	Parallel + non-parallel	–0.3	0.3	0.2	0.1	0.0	0.1	0.1	–0.1	–3.2
		58%	48%	51%	53%	53%	52%	52%	56%	

* Higher number in the cells is the risk in dollars. Lower number in the cells is the percentile.

not measured. This is something that can't be determined without producing the outcomes generated stochastically and comparing the results.[15]

This raises the possibility that governing committees may overestimate the accuracy of their VBIRR measures in their belief that there is little likelihood of results worse than those based upon large parallel rate shocks.

Rate Shocks and EVE-at-Risk

We combine the results from the multiple products, using varying weights, to create two trial balance sheets.[16] We evaluate how the use of rate shocks may impact the measurement and management of EVE-at-risk, a core VBIRR measure. As noted above, shock-based risk measures may understate the actual risks and may mislead governing committees into a confidence that VBIRR has been controlled.

Exhibit 9 shows the two balance sheets. Results are reported in Exhibit 10 and Exhibit 11. In both cases, three-year caps, three-year floors and a five-year swap were chosen to reduce VBIRR. We also include separately the impact of using risk results derived from hedging eight parallel rate shocks plus two nonparallel rate shocks, defined by a steepening and flattening yield curve of +/–150 bps.

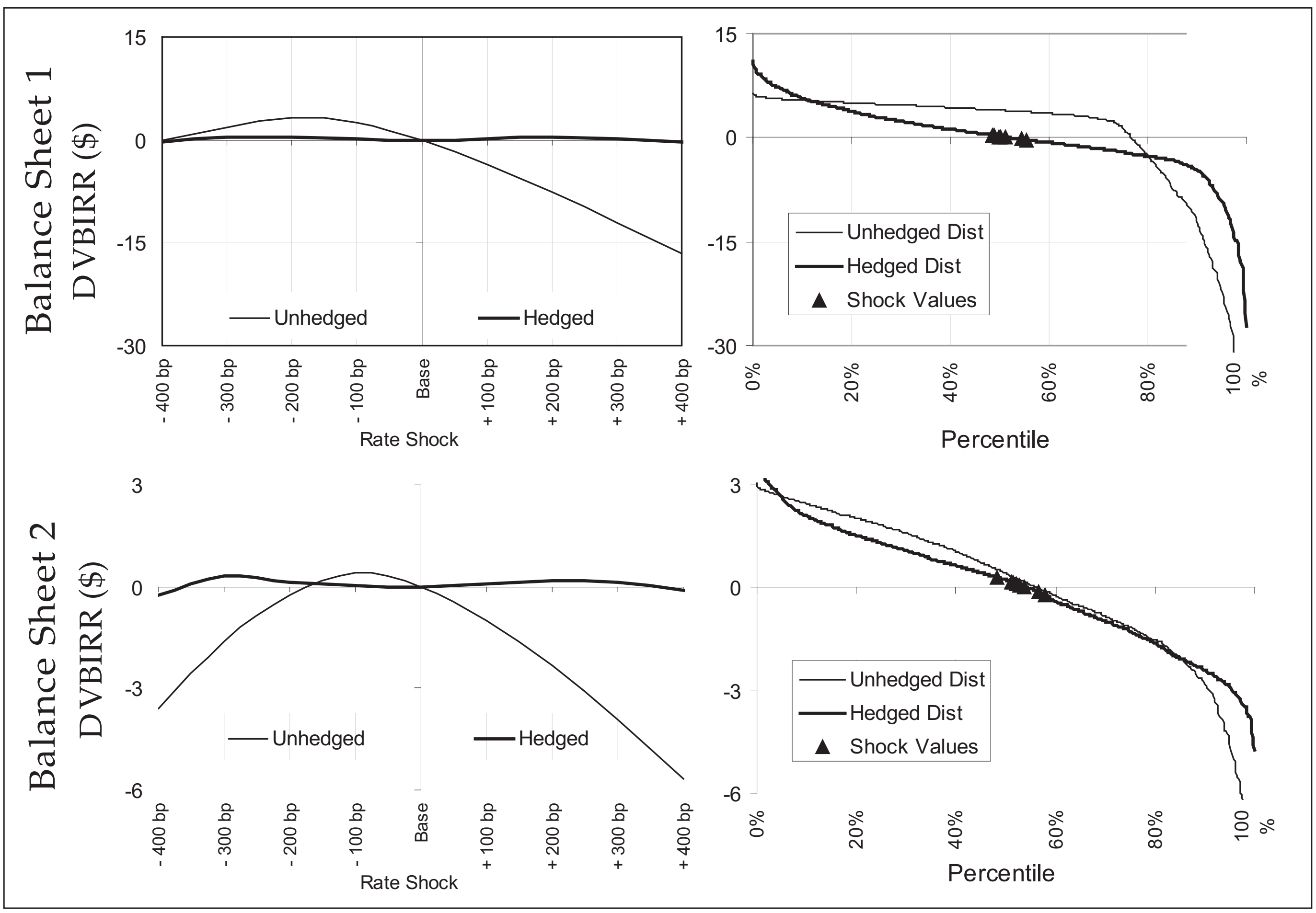
A careful inspection of results in Exhibit 10 and Exhibit 11 shows the following:

- Using results from parallel rate shocks to hedge reduces VBIRR.
- Adding results from two nonparallel rate shocks appears to reduce VBIRR a bit more.
- In both cases, there remains a substantial likelihood of more risk than reported. This observation is not affected by incorporating nonparallel rate shocks.
- Risk at the 97.5th percentile for both balance sheets in the hedged cases is significantly greater than the largest risk measured with rate shocks. This observation is not affected by incorporating the nonparallel rate shocks.

Comparisons with Hedges Designed to Minimize VBIRR in Stochastic Scenarios

The final test in the analysis is to see how much difference it makes to use hedges designed to reduce

Exhibit 11. Unhedged and Hedged Balance Sheets



Note: Hedged distributions are shown for the parallel rate shocks.

VBIRR as measured in the 2,0000 stochastic scenarios. The results in Exhibit 12 clearly support the conclusion that hedging by shocks reduces VBIRR about as well as hedging using all-stochastically generated scenarios. At the same time, hedging the stochastic scenarios harms the performance of hedge results in the shock scenarios.

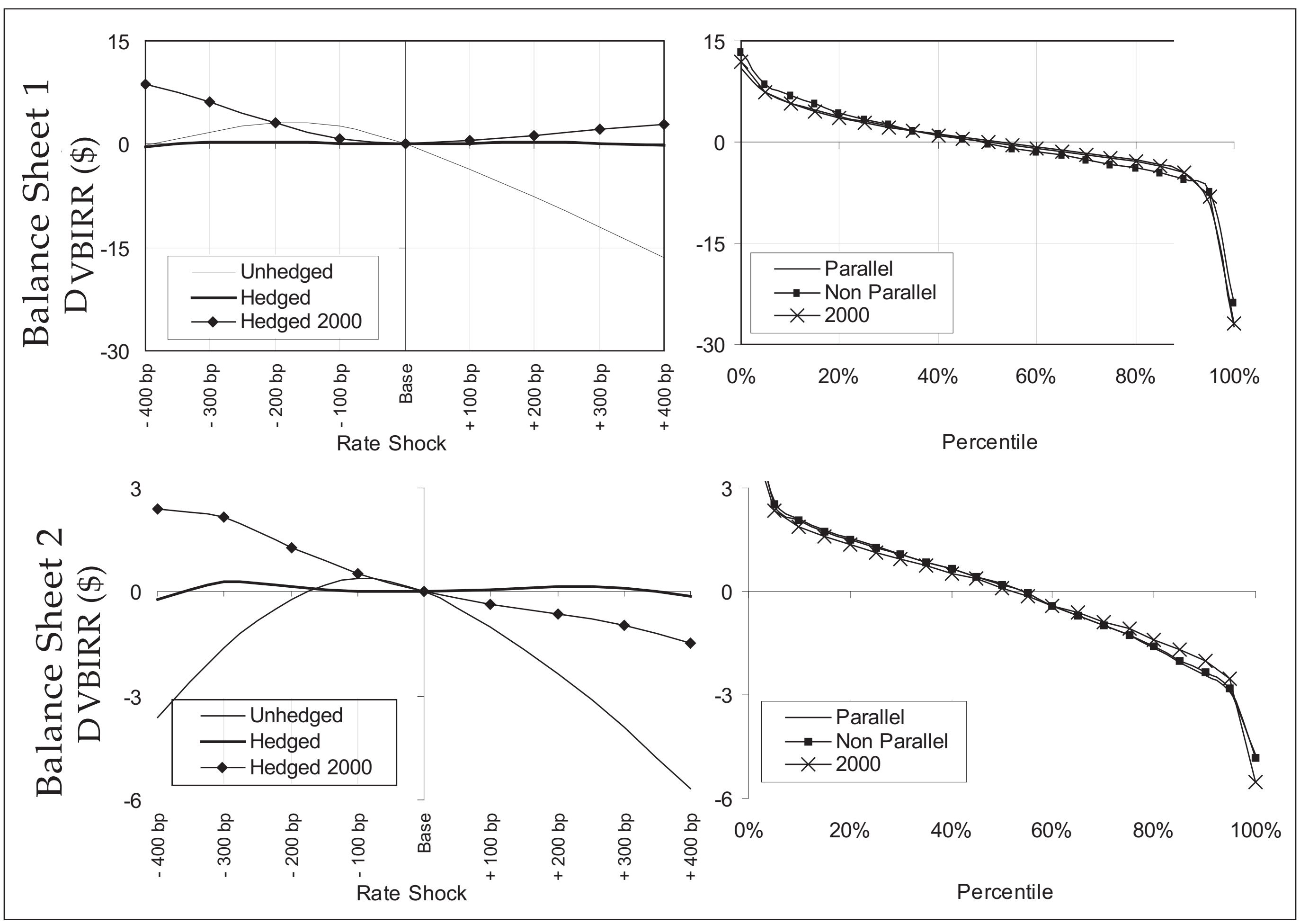
The key finding from these case studies of EVE-at-risk is that rate shocks may be useful proxies for designing hedge solutions but do not provide a reliable measure of how much risk remains. Results from rate shocks, even rate shocks perceived to be of low probability, do not provide a useful measure of how much risk remains or its likelihood of occurring.

A "one shock fits all" … will not provide … consistent information.

Banks Should Test Robustness of VBIRR Measures

Rate-shock methods for hedging VBIRR and EVE-at-risk may be useful for finding hedge solutions but have limited transparency. Large parallel rate shocks and yield-curve shocks, when hedged, result in significant risk reduction but do not easily translate into an understanding of how much risk remains. In all cases, after hedging, we found that only stochastic distributions provided information regarding likelihood and risk at the tails. Indeed, assessment of how well the shocks performed required us to employ a stochastic

Exhibit 12. Hedge Results Compared



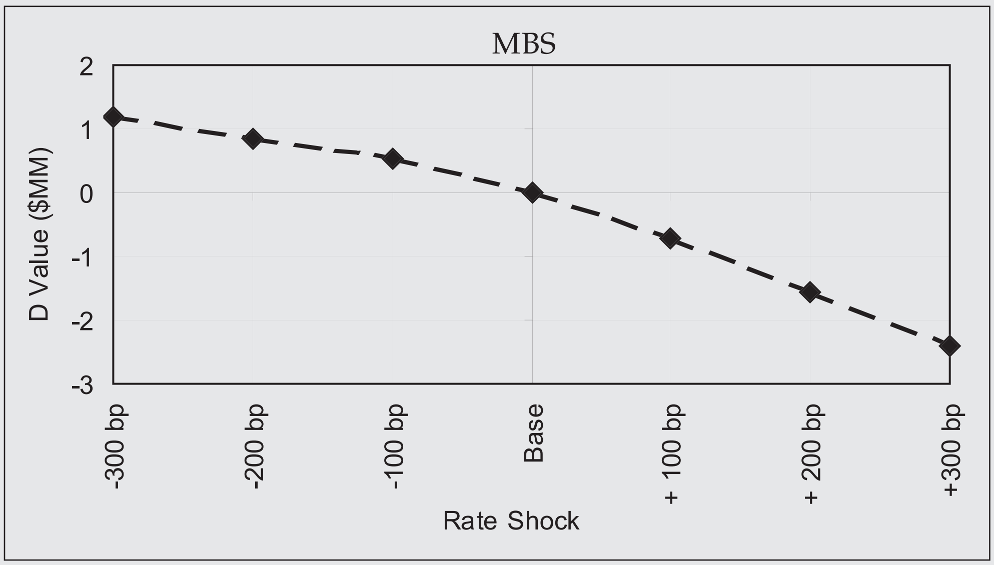
The Four-Step VBIRR Measure

Stochastic-rate-generation technology is required to accurately value financial assets and liabilities containing embedded options. Many banks, mortgage banks and investment portfolio managers have combined rate shocks with stochastic-rate-generation technology to measure VBIRR for financial assets and liabilities containing embedded options. This measurement technique can be summarized as a four-step process:

- Calculate the forward curve and the desired number of shocked yield curves.[1]
- Generate *n* stochastically generated interest paths for each shock case.[2]
- Calculate the NPV of simulated cash flows along each rate path and the average NPV in each shock case.
- Define "risk" as the difference between the average NPV in each shock case relative to the average NPV using the forward curve.

This process is relatively easy to implement in IRR measurement systems. It can be applied to heterogeneous portfolios, such as a bank's balance sheet, and is easily translated into risk payoff graphs, such as the representative risk graph below.

Exhibit 13. Representative Graphical IRR Measure



We have diagramed the IRR associated with the MBS shown in Exhibit 3. The y-axis is based on differences in the NPV of cash flows between the indicated rate shock and the base case. The dashed line is interpolated.

Endnotes

1 Typically, these are parallel shocks of plus and minus 100 bps increments. Some risk analysts use nonparallel shocks and increments other than +/– 100 bps.

2 The precise rules used by risk-measurement systems use different algorithms for generating these stochastic paths.

distribution to see how much risk remained. Consequently, relying on shock-based measures of VBIRR and EVE-at-risk *may* result in significant understatement of risk when compared to risk measures derived from a stochastic risk distribution calibrated to prices in financial markets.

Because balance sheets typically consist of assets and liabilities of various maturities, as well as embedded option structures, the size and type of the rate shock necessary to evaluate the tail risk associated with a net equity position can only be determined empirically. A "one shock fits all" used by many banks will not provide governing committees or regulators with consistent information. Inference of the likelihood of a risk results from the likelihood of a particular rate shock is not possible without further testing.

Banks using models with stochastic generation capability to measure VBIRR and EVE-at-risk and employing the four-step measurement technique employed in this article should analyze the robustness of their risk measures by comparing them to risk using stochastically generated paths calibrated to financial market prices. Given that they are already licensing a stochastic generation system to perform the four-step measure, it is a relatively simple matter to perform these tests. We have found no other way to determine how much VBIRR remains unmeasured without performing these tests empirically.

Regulators who rely on the rate-shock-measurement methodologies to regulate risk may want to consider these results. Rate shocks do not appear to be a consistent guide in understanding the EVE-at-risk in stress-based risk environments. Even when reported risk is within regulatory limits in significant rate shocks, there may remain significant likelihood of far worse results. To the extent regulators are concerned about risks to the insurance funds, they may want to consider techniques that provide more reliable measurement and reporting and, therefore, support more effective hedge strategies.

Rate shocks do not appear to be a consistent guide in understanding the EVE-at-risk in stress-based risk environments.

Endnotes

1 For indefinite-term products, the time horizon is theoretically unlimited.

2 We chose this rate series because it was one of the longer ones available.

3 Michael Arnold and Ray Hawkins, *A Simple Technique to Visualize the Properties of Stochastic Interest Rates*, Bank Asset Liability Management Newsletter, January 2005.

4 *www.federalreserve.gov/releases/h15.*

5 A similar diagram was created for the five-year Treasury rate and is available on request.

6 OCC, *Handbook on Interest Rate Risk* (1997).

7 In this article, we use the term "rate shock" to mean an instantaneous change to a yield curve by a specified amount. We use the term "rate path" to mean an individual rate scenario that is generated stochastically.

8 We used market prices on Libor, swap spreads, caps, floors and swaptions to calibrate the model.

9 The number of financial instruments we evaluated in our analyses is larger than reported in the article. Interested researchers may contact the authors for the results for all 19 instruments evaluated.

10 Technical note on distributional assumptions: The ZM Financial Systems' ALM model has the capability of selecting from a list of recognized rate-generation procedures, including Hull-White (HW, 1993), Black-Derman-Toy (BDT, 1990) and Black-Karasinski (BK, 1991). In this article, we report the results from the BK distributions. For the longer-duration instruments with convexity, the choice makes some difference, because HW distributions have fewer high-rate scenarios. Nine rate shocks were used in the calculations ranging from –400 bps to + 400 bps from the Libor-swap yield curve.

11 There were instances for MBS and hybrids where the –400 bps showed less opportunity than the –300 bps.

12 *www.solver.com.*

13 This value was chosen as representing risk near the tail of the distribution.

14 We define VE as 1-VAR (hedged results)/VAR (unhedged results) for 2,000 scenarios.

15 Alternatively, one could add a panel of nonparallel rate shocks, but this would increase the costs of production.

16 These are not balance sheets in the classical sense. However, we can represent the risk profile of a bank's balance sheet as the sum of repricing mismatches and embedded option effects.